Calculus Assignment

In 1957, the Cambridge astronomer, Fred Hoyle (Sir Fred died in 2001) published a science fiction novel titled **The Black Cloud**. This book is worth a read, even if you don't ordinarily like science fiction, simply because it was written by such an eminent, if somewhat controversial, scientist. In this book, Hoyle tells the story of an intergalactic visit by an intelligent being and the suspense and scientific work that this would generate. This assignment centres on part of the story.

Background:

It is January 1964 and a group of astronomers has been assembled to discuss an odd discovery. An apparently large black cloud seems to be approaching our solar system from somewhere in outer space. After studying a series of photographs and the apparent occlusion of surrounding stars, the scientists have confirmed that the cloud is moving. They have also confirmed that the position of the centre of the cloud is remaining fairly well fixed - indicating that, not only is the cloud moving, but that it is moving directly towards our solar system.

Part of the Story:

One of the astronomers points out that it is important to measure the speed with which the cloud is moving:

"Stars on the fringe of the cloud are partially obscured, ... Their spectrum should show absorption lines due to the cloud, and the Doppler shift will give us the speed."

"Perhaps about a week. It shouldn't be a difficult job."

Another astronomer estimates that the cloud will "be on top of us within fifty or sixty years."

At this point, a youngish, probably handsome, physicist, named Weichart, who obviously knows his calculus, pipes up:

"Sorry I don't understand all this, ... I don't see why you need the speed of the cloud. You can calculate straight away how long the cloud is going to take to reach us. Here, let me do it. My guess is that the answer will turn out at much less than fifty years."

... Weichart left his seat, went to the blackboard ...

"Could we have Jensen's two slides again please?

When Emerson had flashed them up, first one and then the other, Weichart asked: "Could you estimate how much bigger the cloud is in the second slide [which was taken one month later] ?"

"I would say about five percent larger. It may be a little more or less, but certainly not very far away from that," answered Marlowe.

"Right," Weichart continued, "let's begin by defining a few symbols."

Then followed a somewhat lengthy calculation at the end of which Weichart announced:

"And so you see that the black cloud will be here by August 1965, or possibly sooner if some of the present estimates have to be corrected."

Then he stood back from the blackboard, checking through his mathematical argument.

"It certainly looks all right - very straightforward in fact," said Marlowe, putting out great volumes of smoke.

Hoyle adds the following foot note:

The details of Weichart's remarks and work while at the blackboard were as follows:
Write α for the present angular diameter of the cloud, measured in radians,
d for the linear diameter of the cloud,
D for its distance away from us,
V for its velocity of approach,
T for the time required for it to reach the solar system.

To make a start, evidently we have $\alpha = \frac{d}{D}$.

Differentiate this equation with respect to time *t* and we get $\frac{d\alpha}{dt} = \frac{-d}{D^2} \cdot \frac{dD}{dt}$

But $V = \frac{-dD}{dt}$, so we can write $\frac{d\alpha}{dt} = \frac{d}{D^2} \cdot V$.

Also, we have $\frac{D}{V} = T$. Hence we can get rid of *V*, arriving at $\frac{d\alpha}{dt} = \frac{d}{DT}$.

This is turning out easier than I thought. Here's the answer already, $T = \alpha \cdot \frac{dt}{d\alpha}$.

The last step is to approximate $\frac{dt}{d\alpha}$ by finite intervals, $\frac{\Delta t}{\Delta\alpha}$, where Δt is 1 month corresponding to the time difference between Dr. Jensen's two plates; and from what Dr. Marlowe has estimated, $\Delta\alpha$ is about 5 per cent of α, i.e. $\frac{\alpha}{\Delta\alpha} = 20$. Therefore $T = 20 \cdot \Delta t$ or 20 months.

Questions: Examine this bit of mathematics and complete the following:

1. Describe what the variable *d* represents.
2. What is the relationship between *D* and *V*?
3. The first equation, $\alpha = \frac{d}{D}$, requires some explanation. Actually it is an approximation. If you remember your early work with radians, it may help you to understand how Weichart arrived at this equation. Explain how he did so and describe the approximation that he is making.
4. Provide the details of the differentiation resulting in $\frac{d\alpha}{dt} = \frac{-d}{D^2} \cdot \frac{dD}{dt}$.
5. When Weichart states "Also,we have $\frac{D}{V} = T$", what assumption is he making?
6. Explain the details behind the move from $\frac{d\alpha}{dt} = \frac{-d}{D^2} \cdot \frac{dD}{dt}$ to $T = \alpha \cdot \frac{dt}{d\alpha}$. In particular, explain how $\frac{d\alpha}{dt}$ is related to $\frac{dt}{d\alpha}$.
7. "The last step is to approximate $\frac{dt}{d\alpha}$ by finite intervals, $\frac{\Delta t}{\Delta \alpha}$, where Δt = 1 month "
   Using your knowledge of estimation and slopes of tangents, give a graphical illustration of what Weichart is doing here.
8. Explain the statement "$\frac{\alpha}{\Delta \alpha} = 20$, given that $\Delta \alpha$ is about 5% of α".
9. a) Comment on the use of Calculus in this development.
   b) Did you find the application surprising?
   c) How do you feel about the liberal use of soft approximations and hard equations in the physicist's work?
   d) Can you see any points which are perhaps logically suspect?
   e) Why is it not necessary for Weichart to know the actual speed of the cloud?