

## Functions and Word Problems

A. Galileo throws a rock upwards from the top of a tower that is 24m tall. Being a brilliant scientist and an even more brilliant mathematician and after careful experimentation, he has derived the following formulae:  $h = -5t^2 + 7t + 24$  and  $v = -10t + 7$ , where  $h$  is the height of the rock above the ground  $t$  seconds after it was thrown and  $v$  is the velocity at which the rock is travelling  $t$  seconds after it was thrown.

1. Draw a sketch of this situation.
2. Copy and complete the following table in your notebook:

$t$	0		1	1.2				
$v$		0					-14	-23
$h$					24	18		

3. Explain how  $v$  can equal 0. When does this occur (i.e. for what value of  $t$ )? What is the significance of the value of  $h$  at this instance?
4. Describe two things that happen to the rock after the time mentioned in question 3.
5. Explain why the rock is at a height of 24m at two different times. Describe the velocity of the rock at these two instants. Find these two values of  $t$ .
6. In question 5, you found two values of  $t$  which made  $h = 24$ . Describe how these two times relate to question 3.
7. What is the *average* velocity of the rock during the period in which it is falling from its maximum height to the ground? Recall that *average* velocity is determined by the formula  $v_{avg} = \frac{\text{displacement}}{\text{time}}$ .
8. At what time is the rock's instantaneous velocity equal to the average velocity mentioned in question 7?
9. What is the height of the rock at the instant mentioned in question 8?

## Functions and Word Problems (take away)

**B.** Galileo throws a rock upwards from the top of a different tower that is 32m tall. Being a brilliant scientist and an even more brilliant mathematician and after careful experimentation, he has derived these new formulae:  $h = -5t^2 + 12t + 32$  and  $v = -10t + 12$ , where  $h$  is the height of the rock above the ground  $t$  seconds after it was thrown and  $v$  is the velocity at which the rock is travelling  $t$  seconds after it was thrown.

Find the following:

- a) the height of the rock when: a)  $t = 1$  second. b)  $t = 2.5$  seconds.  
the velocity of the rock when: c)  $t = 1$  second. d)  $t = 2.2$  seconds.
- b) When will  $h = 32$ ?  
(i.e. for what value(s) of  $t$  will the rock be 32m above the ground?)
- c) What is the velocity of the rock when  $h = 32$ ?  
(i.e. what is the **initial** velocity of the rock?)
- d) When will the rock hit the ground?  
(hint: what is the value of  $h$  when the rock hits the ground?)
- e) How fast is the rock moving at the instant it strikes the ground?
- f) When will the velocity be 7 m/s? Answer this question for when the rock is going up and when it is going down. Explain the difference in the velocities for these two occurrences.
- g) At what time(s) will the rock be 36m above the ground?
- h) At what time(s) will the rock be 23m above the ground?
- i) At what time does the rock reach its greatest height?
- k) What is the maximum height achieved by the rock?
- l) How high is the rock at the instant when its velocity is -2 m/s?

**CHALLENGE!** If Galileo were to throw the rock upwards from the top of a tower 23m tall, what initial velocity would be required for the rock to go as high as it did in the previous set of questions? How long will the rock take to travel from Galileo's hand to the ground?

## Functions and Word Problems (follow up)

Galileo throws a rock upwards from the top of a tower which is 30m tall. Being a brilliant scientist and an even more brilliant mathematician and after careful experimentation, he has derived the following formulae:  $h = -5t^2 + 25t + 30$   $v = -10t + 25$ , where  $h$  is the height of the rock above the ground and  $v$  is the velocity at which the rock travels  $t$  seconds after it was thrown.

1. a) Find the **height** of the rock when  $t = 1$  s.
  
2. a) Find the **velocity** of the rock when  $t = 1$  s.  
  
b) Find the **velocity** of the rock when  $t = 2.6$  s.
  
3. a) At what time does the rock reach its greatest height?  
  
b) What is the greatest height reached by the rock?
  
4. Find the **height** of the rock when  $v = 5$  m/s.
  
5. What is the velocity of the rock when it is 60 m from the ground?
  
6. What is the velocity of the rock as it hits the ground?
  
7. During a specific one second interval the rock drops a distance of 20 m. What is this interval?