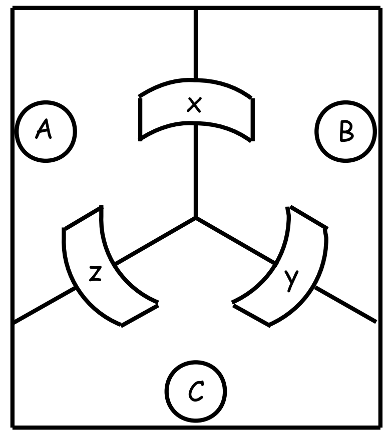
I#12 **Difference**:
The numbers on each bridge is the difference between the numbers in the adjoining fields. Notice that it is not clear if the difference is the result of *A*-*B* or *B*-*A*. This means that there may be multiple solutions.

q.1 (TL)
The solution to this puzzle can be found quite easily.
i) $x = 10 - 6 \therefore x = 4$: ii) $y = 6 - 5 \therefore y = 1$: iii) $z = 10 - 5 \therefore z = 5$

Before we look at the rest of the "difference" puzzles, we will establish a useful result. Consider the puzzle shown on the right.



Reading from field *A*, in a clockwise fashion we get the following:
i) $B = A \pm x$, ii) $C = B \pm y$ and iii) $A = C \pm z$.
Substituting i) into ii) gives us iv) $C = (A \pm x) \pm y$.
Substituting iv) into iii) gives us v) $A = ((A \pm x) \pm y) \pm z$.
Removing the brackets and simplifying the equation gives us:
$0 = \pm x \pm y \pm z$

q.2 (TR)
Using the above result, we get: $0 = \pm 4 \pm x \pm 3$
There are four different ways that this can be true:

1. $0 = +4 \pm x + 3$: i.e. $0 = +4 - 7 + 3$. With this configuration of signs this would generate the following equations: $9 = A + 4$, $C = 9 - 7$ and $A = C + 3$. This means that $(A, x, C) = (5, 7, 2)$.
2. $0 = +4 \pm x - 3$: i.e. $0 = +4 - 1 - 3$, which means that $(A, x, C) = (5, 1, 8)$
3. $0 = -4 \pm x + 3$: i.e. $0 = -4 + 1 + 3$, which means that $(A, x, C) = (13, 1, 10)$
4. $0 = -4 \pm x - 3$: i.e. $0 = -4 + 7 - 3$, which means that $(A, x, C) = (13, 7, 16)$

q.3 (BL)
Again, using the above result, we have $0 = \pm 5 \pm x \pm 4$.

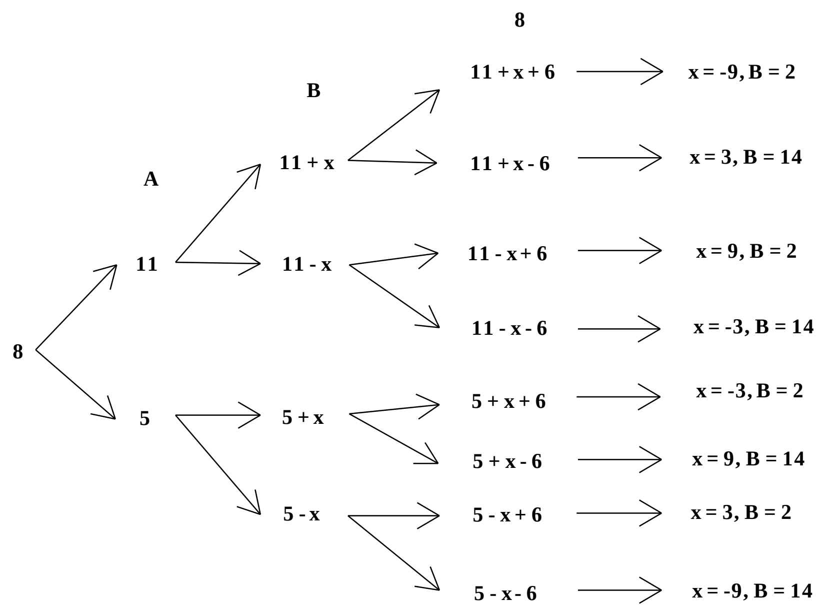
1. $0 = +5 \pm x + 4$: i.e. $0 = +5 - 9 + 4$, which means that $(A, x, B) = (8, 9, -1)$: which is inadmissible.
2. $0 = +5 \pm x - 4$: i.e. $0 = +5 - 1 - 4$, which means that $(A, x, B) = (8, 1, 7)$
3. $0 = -5 \pm x + 4$: i.e. $0 = -5 + 1 + 4$, which means that $(A, x, B) = (-2, 1, -1)$: which is inadmissible.
4. $0 = -5 \pm x - 4$: i.e. $0 = -5 + 9 - 4$, which means that $(A, x, B) = (-2, 9, 7)$: which is inadmissible.

Thus, there is only one admissible solution.

This particular puzzle could be solved more easily if we were to just examine the numbers in the fields directly.

q.4 (BR)
We have $0 = \pm 3 \pm x \pm 6$.
Case 1: $0 = +3 \pm x + 6 \ \therefore x = -9$. $(A, x, B) = (11, 9, 2)$.
Case 2: $0 = +3 \pm x - 6 \ \therefore x = 3$. $(A, x, B) = (11, 3, 14)$.
Case 3: $0 = -3 \pm x + 6 \ \therefore x = -3$. $(A, x, B) = (5, 3, 2)$.
Case 4: $0 = -3 \pm x - 6 \ \therefore x = 9$. $(A, x, B) = (5, 9, 14)$.

Some students (and teachers) may prefer a more visual approach to these questions. There are probably many ways to display this information, but the most effective has to be the one that a student creates for him or herself. As long as they can make sense of their creation and communicate this, the solution is valid. Here is a possible presentation of the information given in the puzzle q4 (BR). Is it clear what is being communicated? How would you present the final solution?



I#13 **Difference**:

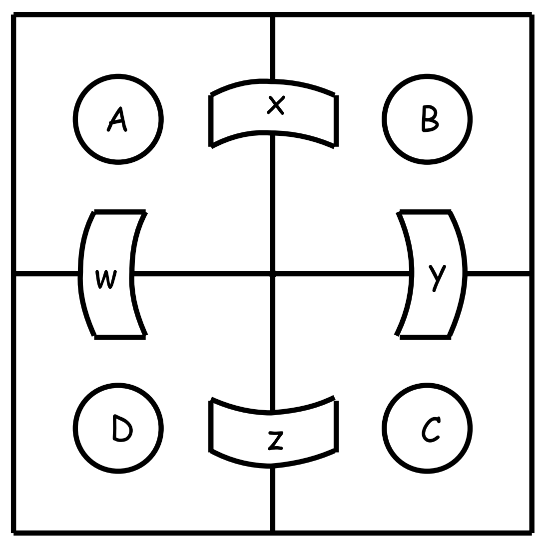
As with the "three field" puzzle, we can consider a general case for the "four-field" puzzle.

Reading from field *A*, in a clockwise fashion we get the following:

i) $B = A \pm x$, ii) $C = B \pm y$, iii) $D = C \pm z$ and iv) $A = D \pm w$.

Following the same steps as before will lead us to...

$0 = \pm x \pm y \pm z \pm w$



If we rearrange this equation as $\pm x \pm z = \pm w \pm y$, we get a result that is curiously similar to the one we found with the "addition" puzzles. Unfortunately there is more than one way to rearrange our new result and the complexity of the signs makes it less useful.

q1(TL)

Since we insist on using only cow-friendly numbers, we can see that *A* has to be 7 larger than 5 i.e. $A = 12$. Also, *B* has to be 8 larger than 5 i.e. $B = 13$. Thus $(x, A, B, y) = (5,12,13,6)$

q2 (TR)

This puzzle is trickier than it looks! Let's first approach it using a more "organic" method.... intelligent guess and check.

Start by letting $A = 1$. Then $B = 10$ and $C = 10$. We will say that $D = 9$. This gives us a total of 30 cows, which is too small. If we increase *A* by 1, this will increase the total by 3. So, in order to increase the total by 12, we should increase *A* by 4. Let $A = 5$, then $B = 14, C = 14$ and $D = 9$. The total is 42. $x = 5$ and $y = 5$.

Let's try another, less "random" approach. We know that *B* and *C* are connected to *A*, but we don't know if they are bigger or smaller. Let's approach this by cases.

1) Consider $A<9$, then $B=A+9$ and $C=A+9$. We know that $A+B+C+9=42$ or $A+B+C=33$. Thus $A+(A+9)+(A+9)=33 \therefore 3A=15 \therefore A=5$. This leads to the answer that we found above.
2) Consider $A>9$. (We can't have $A=9$, why not?) Now $B=A\pm 9$ and $C=A\pm 9$. This means that we have to examine 4 different equations!

$$i)\ A+(A+9)+(A+9)=33 \therefore 3A=15 \therefore A=5, B=14, C=14, x=5, y=5$$

$$ii)\ A+(A+9)+(A-9)=33 \therefore 3A=33 \therefore A=11, B=20, C=2, x=11, y=7$$

$$iii)\ A+(A-9)+(A+9)=33 \therefore 3A=33 \therefore A=11, B=2, C=20, x=7, y=11$$

$$iv)\ A+(A-9)+(A-9)=33 \therefore 3A=51 \therefore A=17, B=8, C=8, x=1, y=1$$

At least this shows that we need to be more careful when we take the "organic" approach!

q3(BL)

We need consider only two cases. $i)\ 0=5-6-7+8$ and $ii)\ 0=-5+6+7-8$.

i)Start with $B=14$ (can you see why?) then $C=8$, $D=1$ and $A=9$ for a total of 32. This is 12 too small, so our solution is $B=17$, $C=11$, $D=4$ and $A=12$.

ii) Start with $D=14$ (again, can you see why?) then $A=6$, $B=1$ and $C=7$ for a total of 28. This is 16 too small, so our solution is $D=18$, $A=10$, $B=5$ and $C=11$.

q4(BR)

We have actually already solved this puzzle in q3.

I#14 **Difference**:

q1(TL)

This puzzle is not possible since $0 \neq \pm 4 \pm 1 \pm 2$.

q2(TR)

We have two cases $i)\ 0=11-5-6$ and $ii)\ 0=-11+5+6$

i) Smallest possible value for *B* is 12. Thus $C=7$ and $A=1$, for a total of 20.

ii) Smallest possible value of *A* is 12. Thus $B=1$ and $C=6$, for a total of 19, which is the smallest total.

q3(BL)

We have two cases $i)\ 0=5+4-3-6$ and $ii)\ 0=-5-4+3+6$

i) Smallest possible value for *C* is 10. Thus $D=7$, $A=1$ and $B=6$, for a total of 24.

ii) Smallest possible value of *A* is 10. Thus $B=5$, $C=1$ and $D=4$, for a total of 20, which is the smallest total.

Why don't we ask for the largest possible total?

q4(BR)

Since $0 \neq \pm 4 \pm 5 \pm 7 \pm 3$, this puzzle is not possible.

I#15 **Difference**:

q1(top)

We have $0 = \pm 7 \pm 5 \pm x$. This means that 7 and 5 will either have the same sign (in which case $x = 12$) or they will have different signs (in which case $x = 2$). You should note that "the difference between 5 and 7 is 2" and that the "difference between 7 and 5 is 2".

q2(bottom)

We have our work cut for us here!

We have $0 = \pm 5 \pm 8 \pm 4 \pm x$. We have four cases to examine:

$i)\ 0 = 5 + 8 + 4 \pm x,\ x = \pm 17$

$ii) 0 = 5 + 8 - 4 \pm x,\ x = \pm 9$

$iii) 0 = 5 - 8 + 4 \pm x,\ x = \pm 1$

$iv) 0 = -5 + 8 + 4 \pm x,\ x = \pm 7$

You should note that case *i*) is the same as $0 = -5 - 8 - 4 \pm x$. Using this idea we can reduce the number of possible cases from 8 to 4.

i) *a*) $x = -17,\ D = 18, A = 1, B = 6, C = 14$ for a total of 39
b) $x = 17,\ A = 18, B = 13, C = 5, D = 1$ for a total of 37

ii) *a*) $x = -9,\ D = 10, A = 1, B = 6, C = 14$ for a total of 31
b) $x = 9,\ A = 14, B = 9, C = 1, D = 5$ for a total of 29

iii) *a*) $x = -1,\ B = 9, C = 1, D = 5, A = 4$ for a total of 19
b) $x = 1,\ A = 6, B = 1, C = 9, D = 5$ for a total of 21

iv) *a*) $x = -7,\ D = 13, A = 6, B = 1, C = 9$ for a total of 29
b) $x = 7,\ B = 13, C = 5, D = 1, A = 8$ for a total of 27

Case *iii*) *a*) gives the smallest total.