GETTING A “FEEL” FOR MATHEMATICS

How does one get a ***feel*** for mathematics? Is this something that can be taught or engendered? Does it evolve with experience? Can it be nurtured?

Teachers will often talk about a ***mathematical intuition*** or a ***mathematical sense*** and how some students seem to have it and, alas, how too many students don’t. Can it be itemized in a strand or described as an expectation? Can it be taught, learned or assessed? Where does one find it in the curriculum?

There is no doubt that some students find it easy to internalize certain mathematical ideas. They effortlessly make connections between familiar concepts and novel situations. You hear them say things like “ah, that’s just like such and such” or “don’t you remember when we did so and so?” These students, truly, have a firm grasp on what they have learned. They seem to have some sort of “prior” knowledge or previous familiarity.

But “grasp” and “feel” are tactile words. For these students, mathematical concepts are quite real – even tangible. Their learning is remembered like a physical event. Concepts become part of their vocabulary and life-experience. Mathematical ideas form a lens through which they view their world.

As mathematics teachers, we want to reach all of our students in such a significant and purposeful way. They should all find mathematics real, useful and engaging. Surely, they should experience mathematics in a way that is similar to their enjoyment of music and absorption in art.

As students “do” mathematics, there should be a seamless interlacing of internal and external events. Having them describe physical attributes and relationships using mathematical vocabulary and ideas, while simultaneously manipulating concrete objects and conceptual symbols, is a great way to foster this connection.

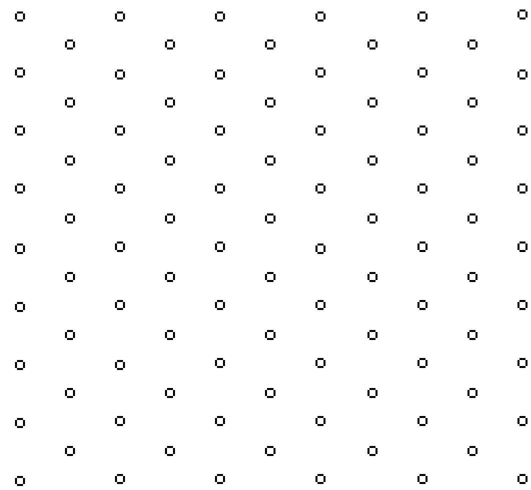
For a long time now, many teachers of our younger children have been well aware of the benefits of using manipulatives. Cubes, Cuisenaire rods and attribute blocks are commonplace in many elementary school classrooms. Thousands of students have been introduced to geometric shapes and relationships using pegs and elastic bands. They have come to recognize patterns and sequences in both numbers and physical models.

A few secondary school teachers have been using similar aids in their classes for years – but, by and large, they have been keeping this a secret. It is unfortunate, but many senior (and intermediate!) mathematics teachers scoff at the use of small plastic cubes and such, as being too juvenile – somehow beneath the dignity of the discipline.

I hope to convince you, with this resource, that rather than being juvenile, small plastic cubes can actually bring challenging and rewarding experiences to your classroom. I also hope to convince you that by “concretizing” the learning, many more of your students will be able to “grasp” some of those elusive, abstract concepts and perhaps even get a “feel” for the processes.

This resource focuses on the use of ‘cubes’, one of many forms of manipulatives that are commercially available for your mathematics classroom. You will notice that the manipulatives are not used to merely represent mathematical concepts. I have tried to utilize them as the focus or object of mathematical investigation.

READING ISOMETRIC DRAWINGS

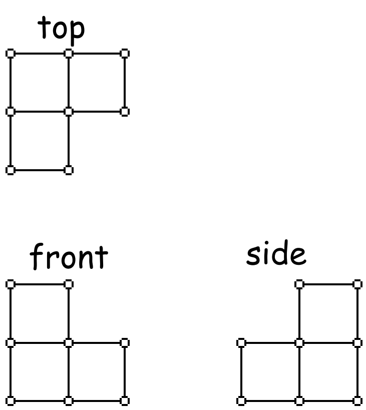


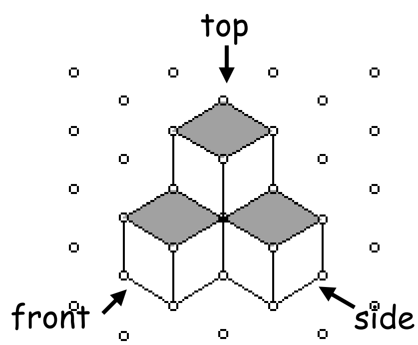
This is a piece of isometric dot paper. You will notice that the dots do not line up as they would on regular graph paper. Instead, they are offset at 60º.

This isometric grid allows us to create "three-sided" views.

Consider the diagrams below. In the first picture you will see the familiar "top-front-side" views of a small model built from cubes.

In the second picture you will see the same model in a "three-sided" view sketched on isometric dot paper. The upper surfaces have been shaded to help you visualize what the model looks like.





By looking at the "top-front-side" views try, to visualize the model. By considering the "three-sided" view, again, try to visualize the model.

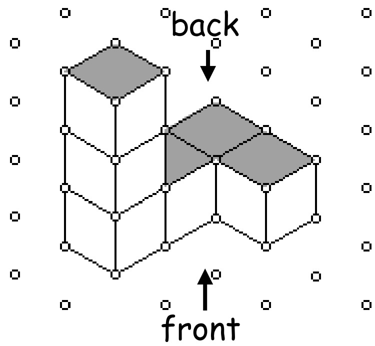
Compare the way that you perceive the model by looking at the two sketches.

BUILD the illustrated model using cubes. Check with a classmate to see if your model is correct.

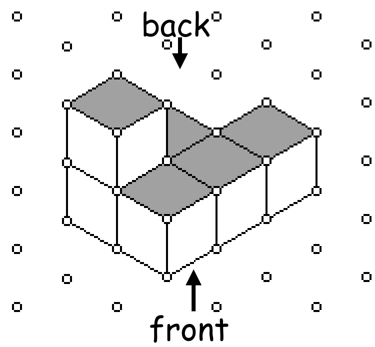
1. Each of these two models contains 6 cubes.

Build the models with cubes and check with a classmate to see if they are correct.

a)

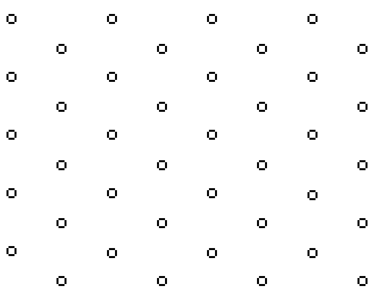


b)

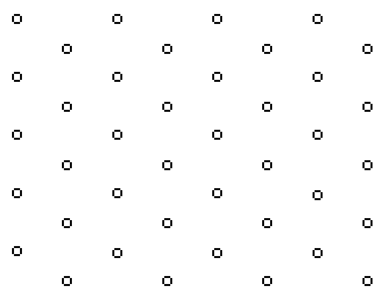


2. **Sketch each of these models**, as they would appear from the back.

a)



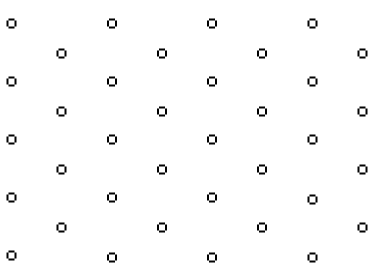
b)



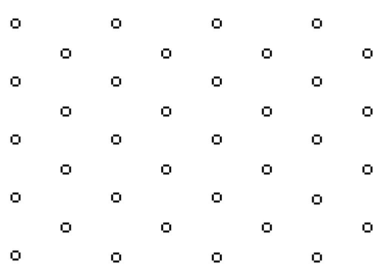
3. Using 6 cubes build your own model and ask your classmate to sketch an isometric drawing.

Try sketching your model from a different view.

a)

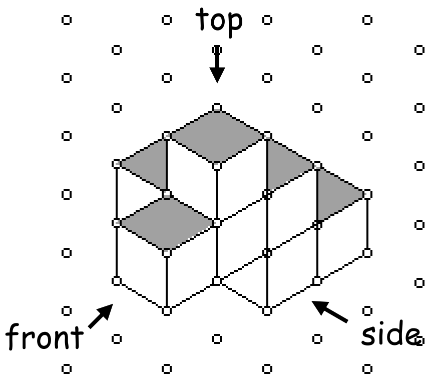


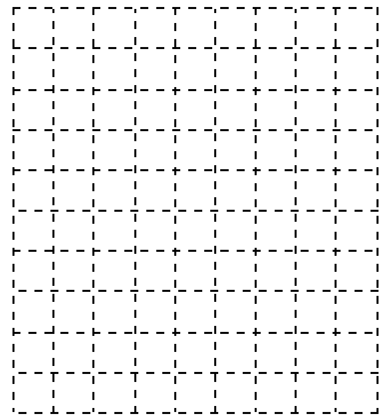
b)



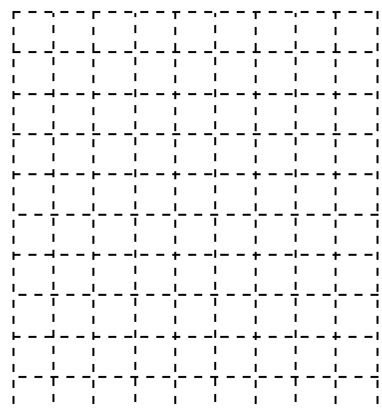
HIDDEN PIECES

1. a) Using only 7 cubes, **build the following model**.
b) **Draw the top, front and side views** of this model.

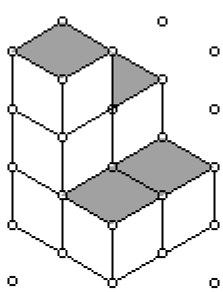


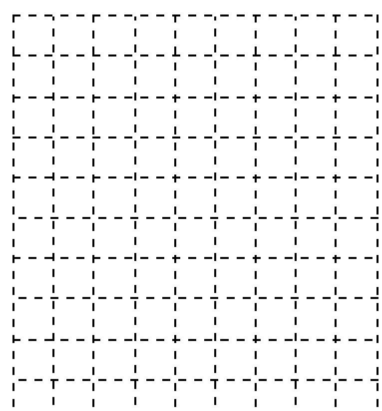


2. a) Using 12 cubes, **build the above model.**
b) **Draw the top, front and side views** of your model.

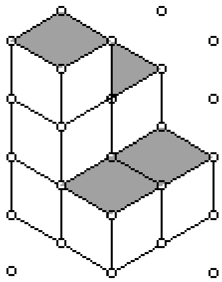


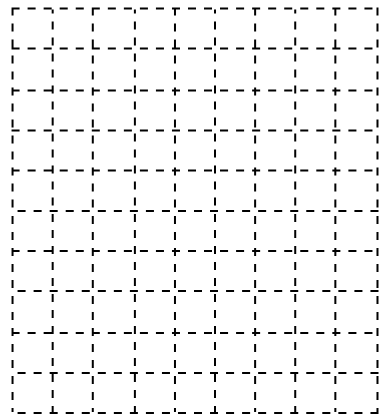
3. a) **Build this model** using 7 cubes.
b) **Draw the top, front and side views** of your model.



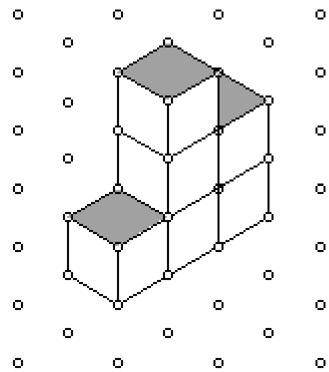


4. a) **Build this model** using 11 cubes.
b) **Draw the top, front and side views** of your model.
c) **Compare** your drawings with those of a classmate.
d) **Comment** on any differences that you see.

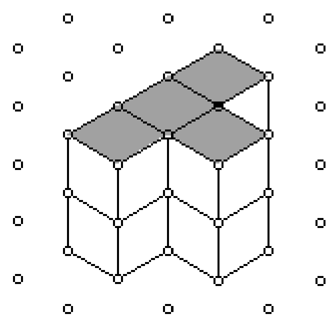




5. a) What is the minimum number of cubes that could be in this model?
b) What is the maximum number of cubes that could be in this model?

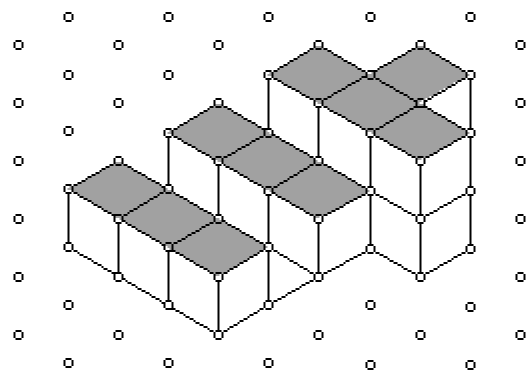


6. a) What is the minimum number of cubes that could be in this model?
b) What is the maximum number of cubes that could be in this model?



Questions 2, 3, 4, 5 and 6 introduce one of the difficulties with isometric drawings. It is possible to hide cubes. The model in question 6, could contain as few as 6 cubes – just what you see. But there is actually no limit to the number of cubes that it might contain! How many did you say could be in the model?

7. Consider this view from the back of the model illustrated in question 6.

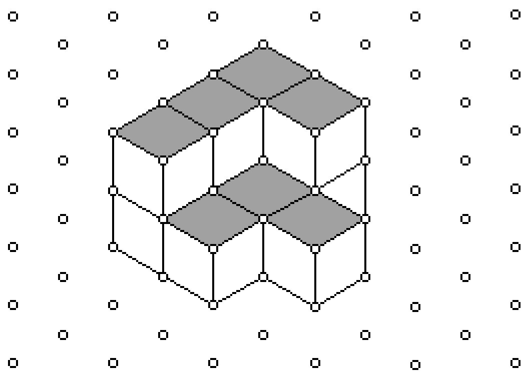


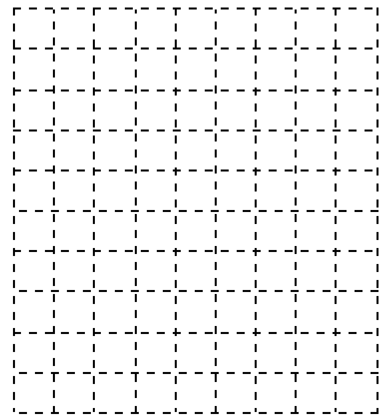
Build this model and convince yourself that if you hold it in just the right orientation, it will look exactly like the above model in question 6. Do you see how you could add 5 more cubes to this model and still not be able to see them from the front? Could you add 5 more?

None of the illustrations that you will encounter in this resource will contain any “tricks” like this - unless specifically indicated. From now on, the back view of a model will not reveal any “surprises”. You will be able to count the number of cubes in a drawing and be comfortable with your answer.

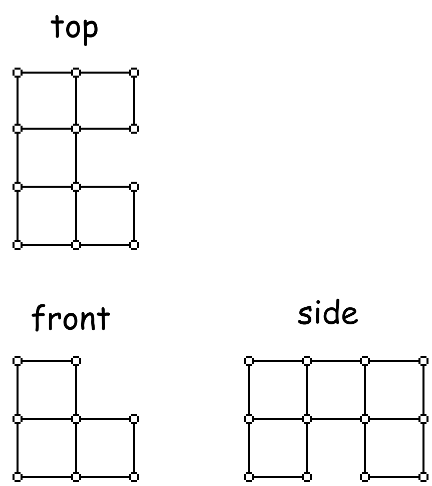
DIFFERENT WAYS OF LOOKING AT THINGS

1. **Build this model** using 11 cubes and then **sketch the top, front and side views**.

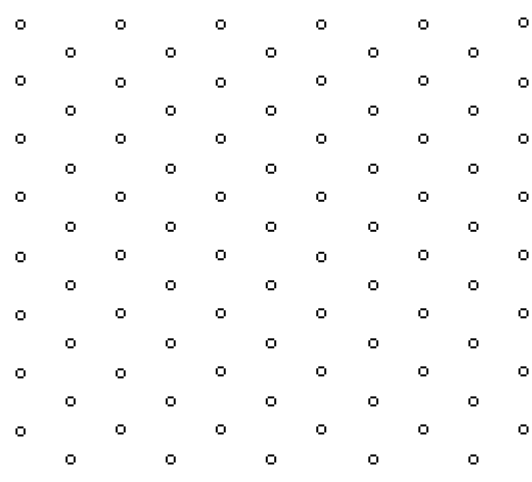




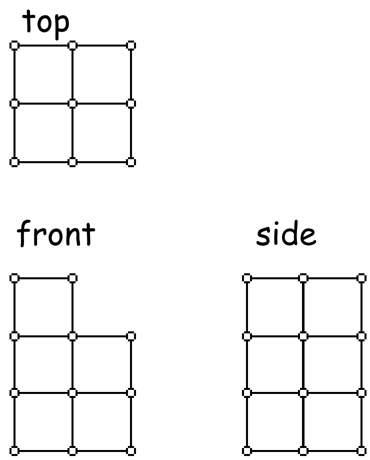
2. a) Using cubes, **build a model** of the figure shown in the following "top-front-side" view.



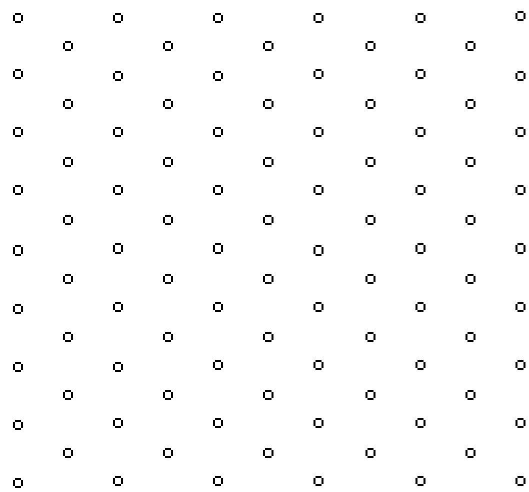
b) **Sketch an isometric drawing** of your model



3. a) Using cubes**, build a model of the figure** shown in the following “top-front-side” view. There may be more than one possible answer.



b) **Sketch an isometric drawing** of your model.



b) What is the minimum number of cubes that could possibly be in this model?

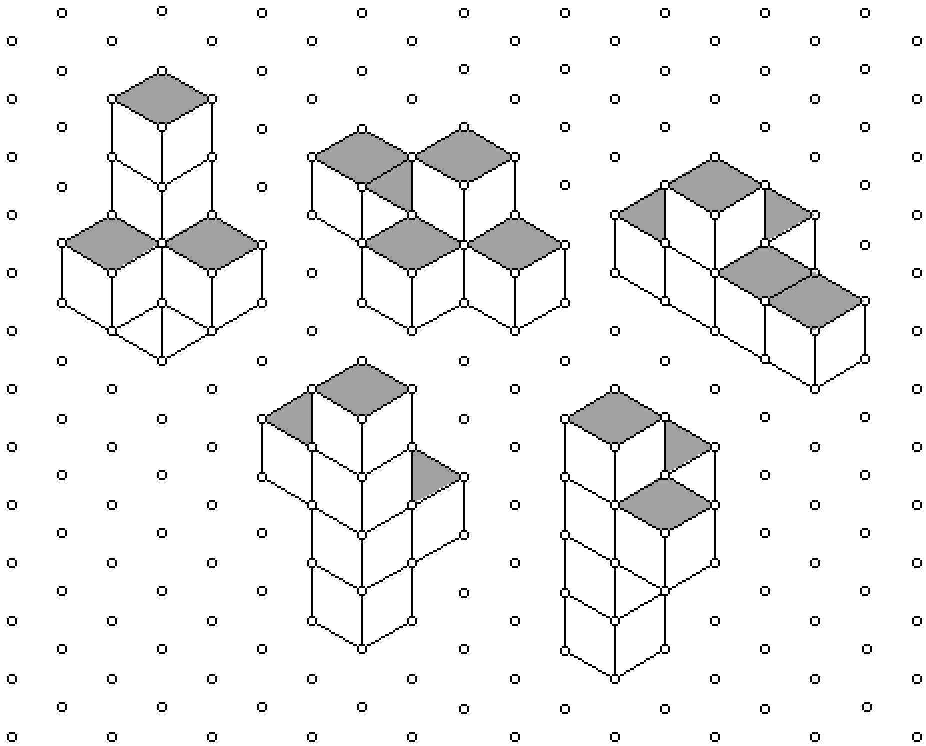
c) What is the maximum number of cubes that could possibly be in this model?

d) The problem of hidden cubes is not so serious with “front-side-top” drawings. **Explain** what this statement means. Is it true? **Explain your thinking**.

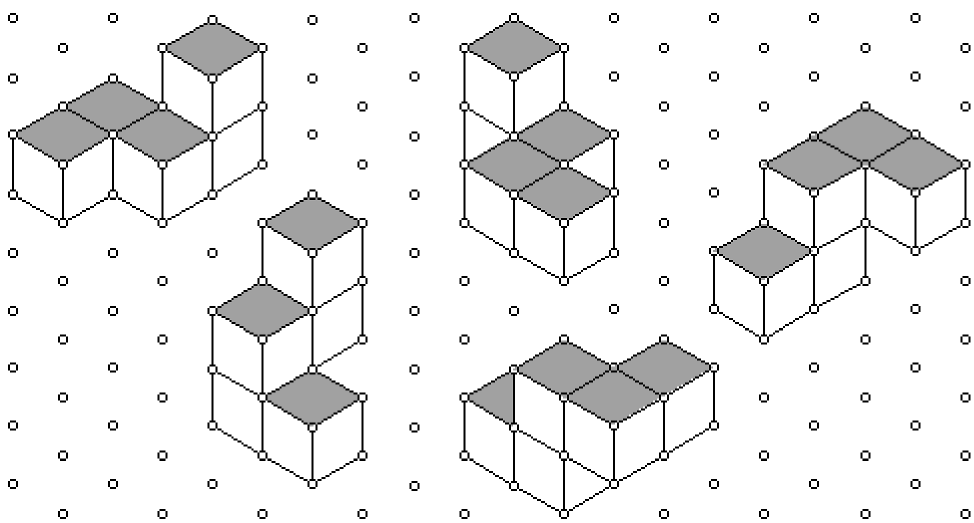
e) Check with a drafting teacher to see how the problem of hidden parts is handled when using proper drafting techniques.

ODD BLOCK OUT

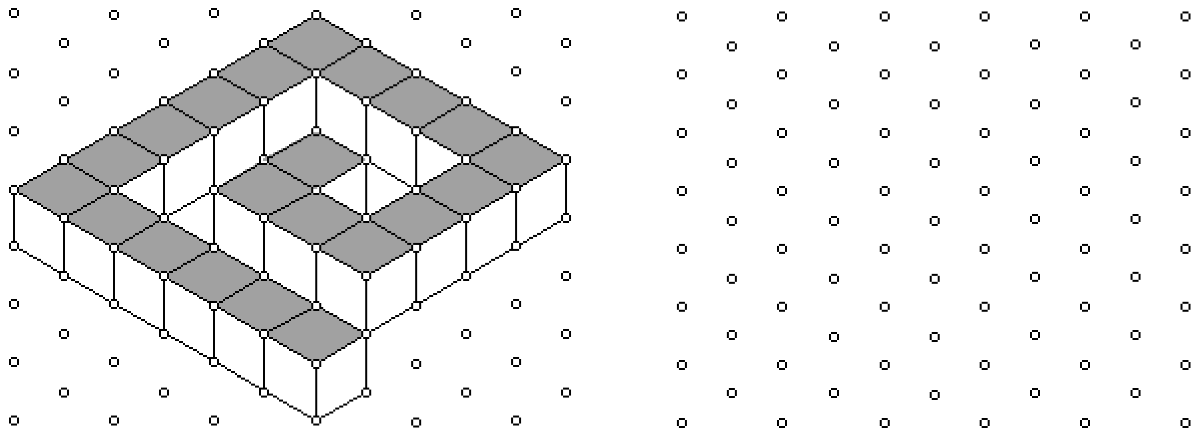
1. a) Which of the following models doesn't belong?
b) Explain why the odd-one-out doesn't fit in.



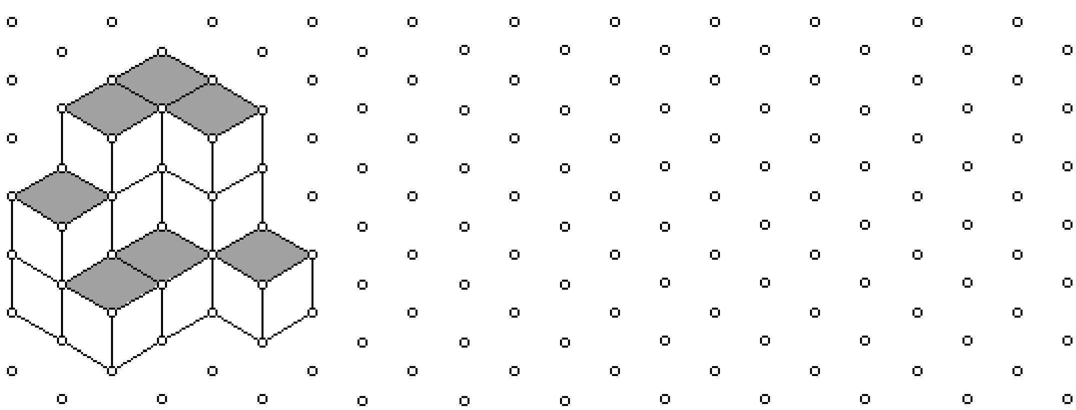
2. a) Which of the following models doesn't belong?
b) Explain why the odd-one-out doesn't fit in.



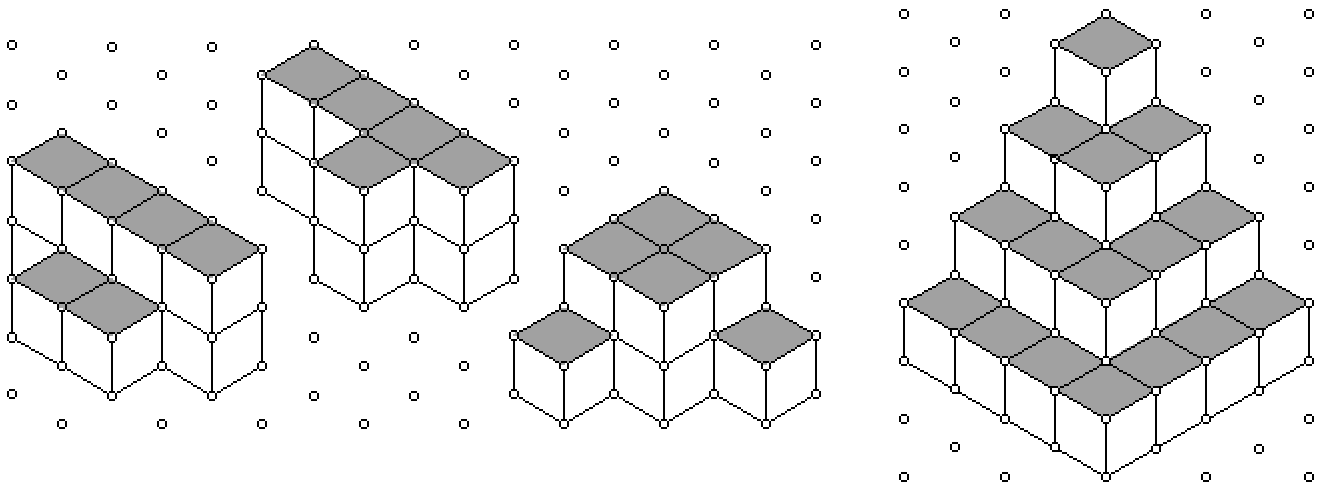
3. a) **Build a model** of the piece that you would need to complete a 6x6 square.
f) **Draw an isometric picture** of the piece that you would need to complete a 6x6 square.



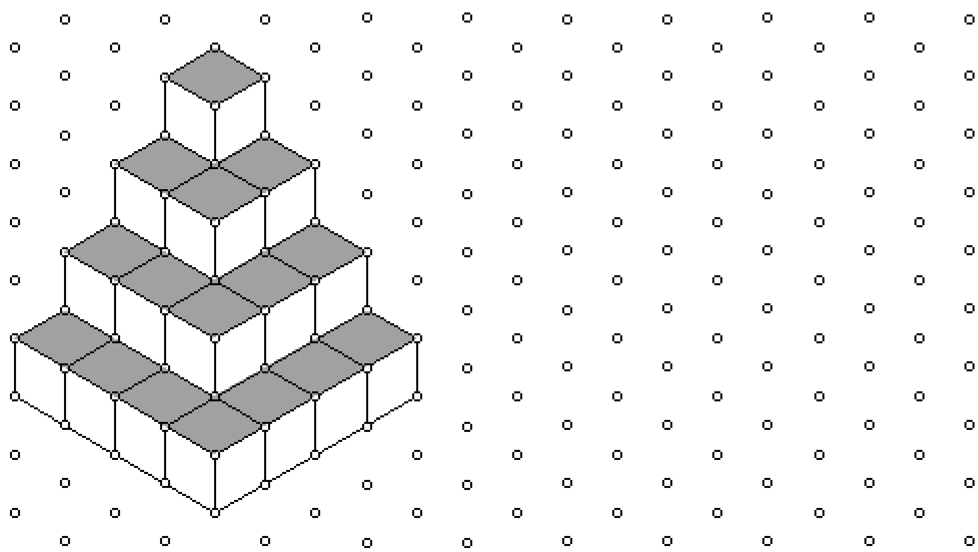
4. a) **Build a model** of the piece that you would need to complete a 3x3x3 cube.
b) **Draw an isometric picture** of the piece that you would need to complete a 3x3x3 cube.



5. a) **Build these three pieces and fit them together** to form the pyramid shown below on the right.



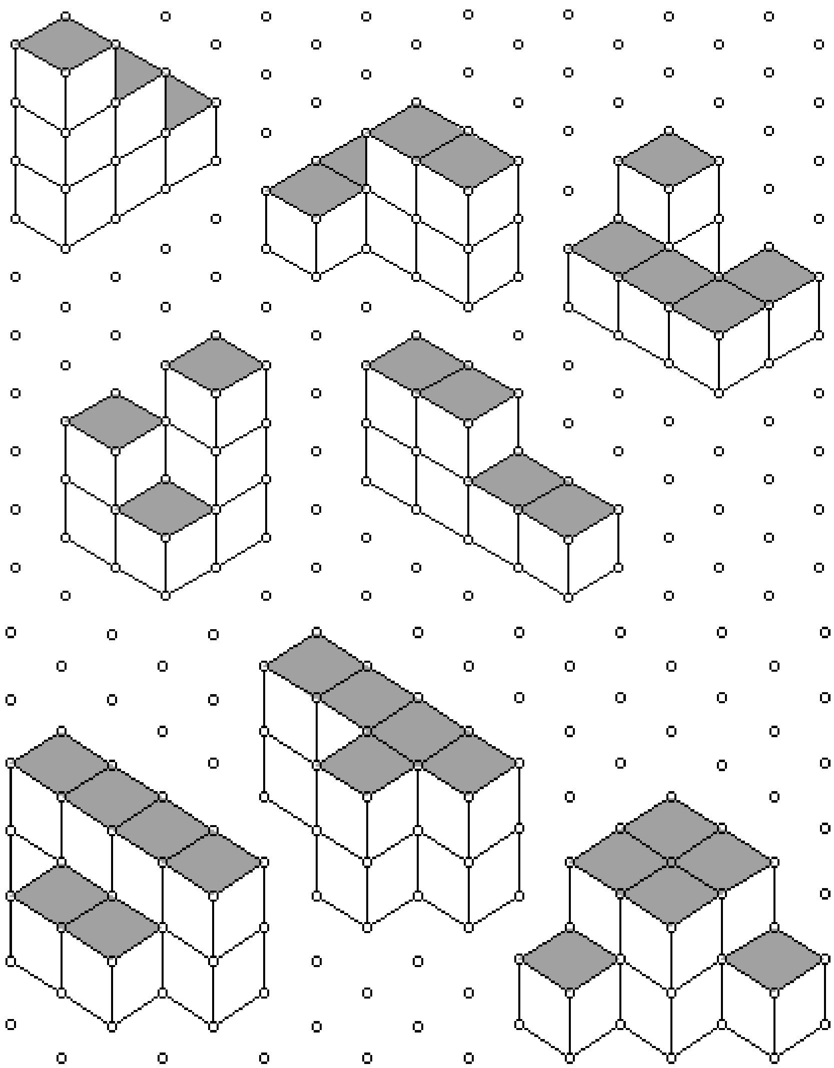
6. a) **Build a model** of the piece that you would need to complete a 4x4x4 cube.
b) **Draw an isometric picture** of the piece that you would need to complete a 4x4x4 cube.



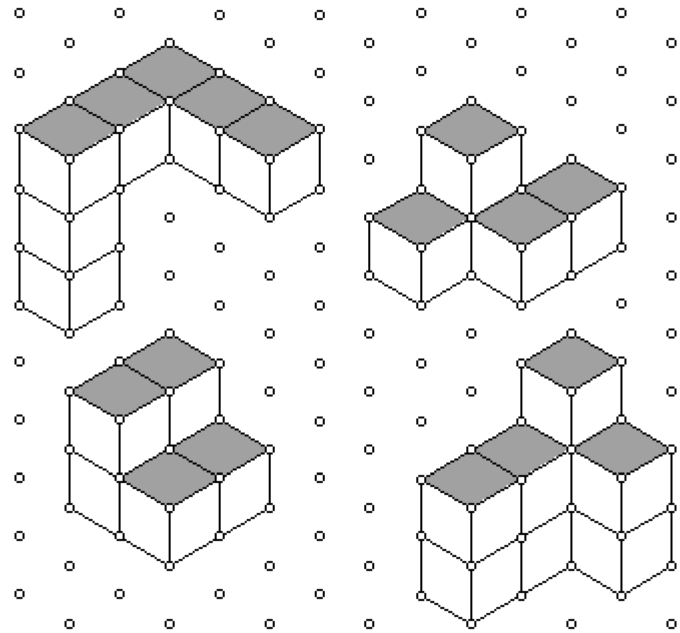
7.

Think…

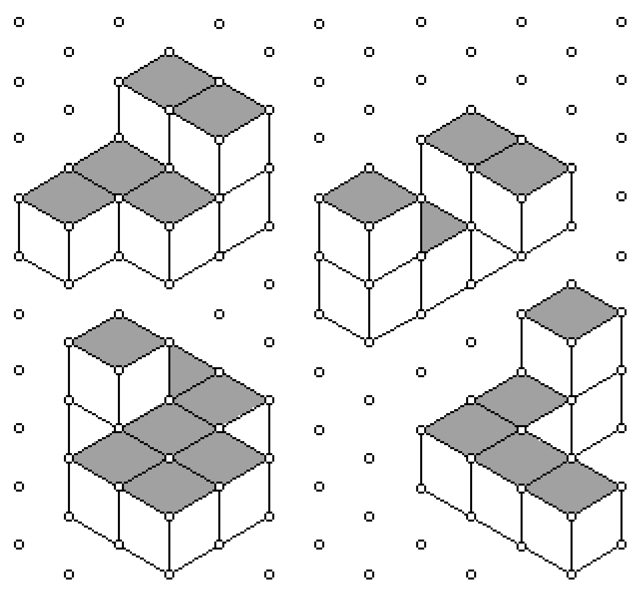
Each of the five models shown below contains 6 cubes. Build the models and fit them together to form the pyramid shown below.



8. Build all of these models. **Fit the four pieces** together to form a solid 3x3x3 cube.

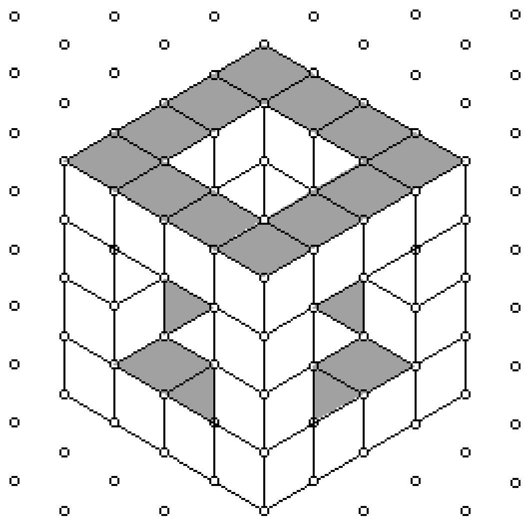


9. **Build** all of these models. **Fit the four pieces together** to form a solid 3x3x3 cube.



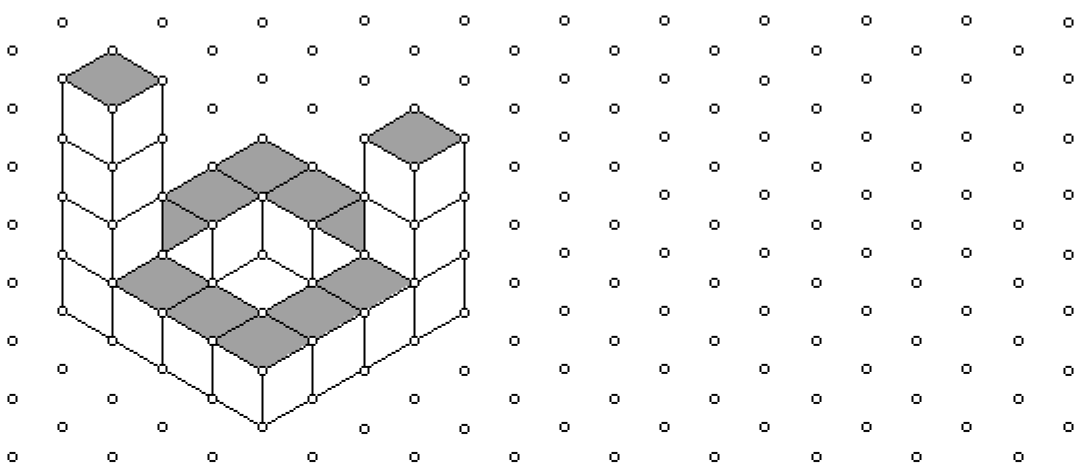
10. This is a 4x4 skeleton cube.

a) How many cubes would you need to build it? **Build a model** of this figure.



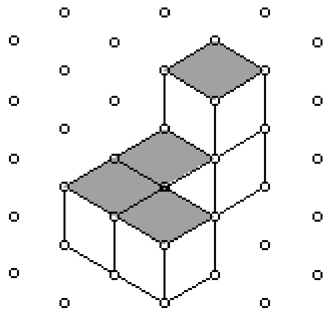
b) A partial 4x4x4 skeleton cube is shown below. **Build the piece** that is needed to complete the model.

c) **Draw an isometric picture** of the piece that you would need to complete the 4x4x4 skeleton cube.

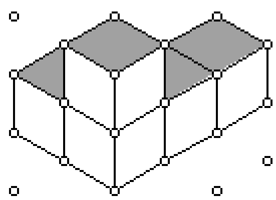


PUZZLING CUBES

1. **Build** four copies of this model. **Fit the four pieces together** to form a 3x3x3 skeleton cube.



2. **Build four copies** of this model. **Fit the four pieces together** to form a 3x3x3 skeleton cube.



3. **Create 4 congruent models** of your own design that will fit together to form a 3x3x3 skeleton cube.

4.

Think…

a) How many cubes will there be in each of four congruent models that go together to form a 5x5x5 skeleton cube?

b) Design your own set of four congruent pieces that will fit together to form a 5x5x5 skeleton cube.

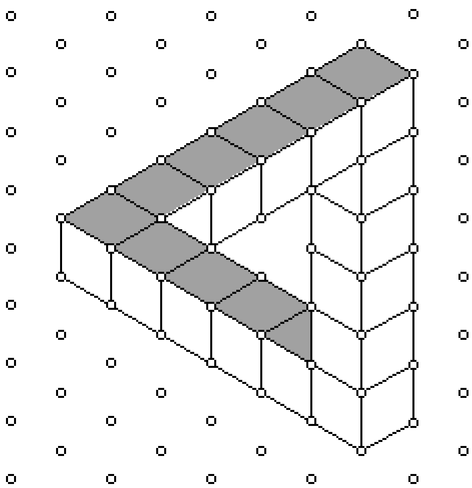
c) Can you create a different way to do this?

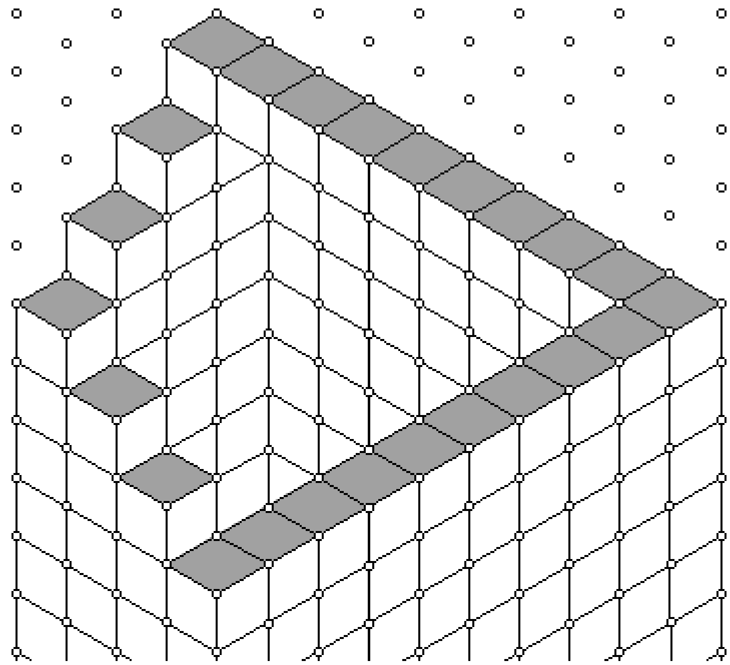
F. ***IMPOSSIBLE FIGURES***

The following illustration shows two figures that are impossible to build. You can try to build them if you like, but don't be disappointed!

Explain why the figure on the right is impossible.

How many cubes would it take to build the model on the left (even if you cannot actually build it!)?





M.C. Escher, the great Dutch graphic artist, used impossible figures in many of his drawings. If you are not familiar with his work, you might enjoy a trip to the library where you will find many examples of his breathtaking and perplexing art. The impossible figure on the right was originally described by the English mathematician Roger Penrose.

Why is it that we can draw these figures using isometric dot paper and yet the model couldn't possibly exist in the real world? **Try to explain** what it is about isometric dot paper that allows us to draw apparently realistic pictures of unrealistic objects. You might consider ideas such as one and two point perspective and vanishing points.

Try to draw a fantastic creation of your own. Share your creation and ideas with an art teacher and your classmates.

NOTES TO THE TEACHER

The following suggestions may help make the introduction of manipulatives into your mathematics class an enjoyable and rewarding experience for all concerned.

1. Introduce the cubes to your students in a reasonably formal manner. The cubes are manipulatives that are being used to assist with the understanding of mathematical concepts. The students are to be encouraged to treat these as a serious visualization tool.

2. Students must be given an opportunity to become acquainted with the cubes. Isometric drawings offer a number of anomalies that may cause perceptual difficulties. Give the students a chance to play. Let them explore and experiment with drawing models. You can challenge them to create ever more complex models and drawings. This first section of the document has been designed, primarily with this in mind.

3. Establish classroom management procedures with respect to the use of cubes. You should devise a distribution scheme and you will need to guarantee that all of the cubes have been returned at the end of the class. One suggestion is to have students return the cubes in bundles of "sticks" that are just long enough to fit neatly in a storage box. A visual check is preferable to an accounting approach! You may wish to use labeled tubs, zip-lock bags or simple cardboard boxes.

4. Activities should not be limited to the exercises described in this document. You will no doubt discover your own opportunities for using cubes. Students should be encouraged to explore ways of modelling problems using these and other manipulatives. The uses of cubes described in this document are probably too limiting. Be on the lookout for interesting, new and varied activities.

5. One of the main objectives of this document is to provide classroom teachers and their students, opportunities to see and feel mathematical concepts. Students are urged to work with cubes, graphing calculators, isometric dot paper and regular pencil and paper. It is hoped that they will effortlessly shift their focus from one representation to another.

Even though some students will be able to work directly from the pictures, they should be encouraged to engage in the actual construction of the models. Hopefully, they will start to look at the cubes and number patterns through algebraic and graphical eyes.