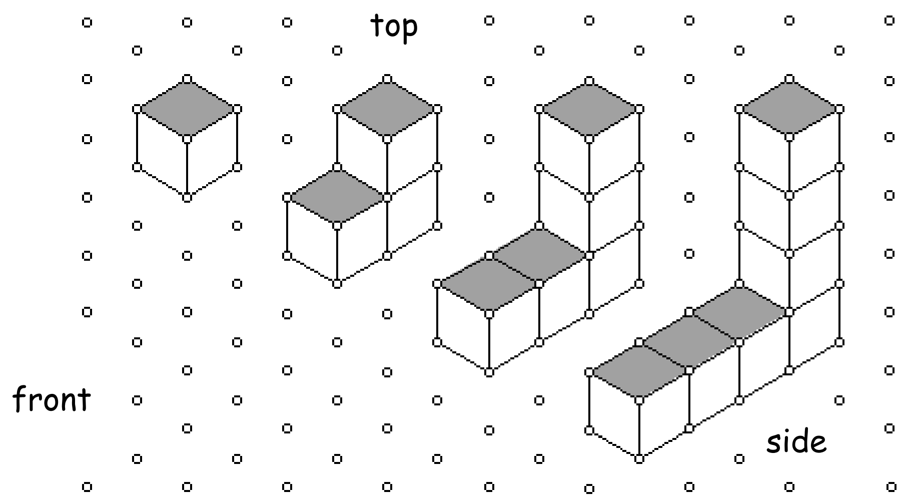
LINEAR MODELS – PATTERNS AND ALGEBRA

Now that you are familiar with the way cubes look on the table and on isometric dot paper, we can begin to look at some patterns – both physical and numerical.

A. ***GROWING "L'S"***

In this sequence, you can see how each model has been formed by simply adding 2 cubes to the previous model. One cube is attached to the front face while the second cube is attached to the top face. Each model contains 2 cubes more than its predecessor. Build this sequence of models using cubes.



When we look at the actual number of cubes in each model, we see a similar pattern emerging.

Model Number	1	2	3	4
Number of Cubes	1	$1+2$ $=1+2$ $=1+(1)\times 2$ $=3$	$3+2$ $=1+2+2$ $=1+(2)\times 2$ $=5$	$5+2$ $=1+2+2+2$ $=1+(3)\times 2$ $=7$

a) How many cubes will there be in the 5th, 6th and 7th models?

b) In the row "Number of Cubes", you will see the sequence 1, $1+2$, $3+2$ and $5+2$. **Explain** how these expressions are connected to the above sequence of models.

c) In the row "Number of Cubes", you will see the sequence 1, $1+2$, $1+2+2$ and $1+2+2+2$. **Explain** how these expressions are connected to the above sequence of models.

d) In the row "Number of Cubes", how are the numbers in the brackets related to the Model Number?

e) What number would be in the brackets for the 20th model?
What expression would be in the brackets for the n^{th} model?
How many cubes would there be in the 45th model?

f) The number of cubes in the n^{th} model is given by the formula:

$$N = 1 + (n-1) \times 2 .$$

Here, N represents the total number of cubes in the n^{th} model.

If you look carefully, you should see that this formula reflects the way that the models shown above were actually constructed.

For the first model, $n = 1$. We can see that there is just 1 cube. If we replace n with 1 in the formula, we get:

$$N = 1 + (1-1) \times 2$$
$$N = 1$$

For the second model: $n = 2$. If we replace n with 2 in the formula, we see how the single cube in the first model is increased by 2 new cubes to form the second model:

$$N = 1 + (2-1) \times 2$$
$$N = 1 + (1) \times 2$$
$$N = 1 + 2$$

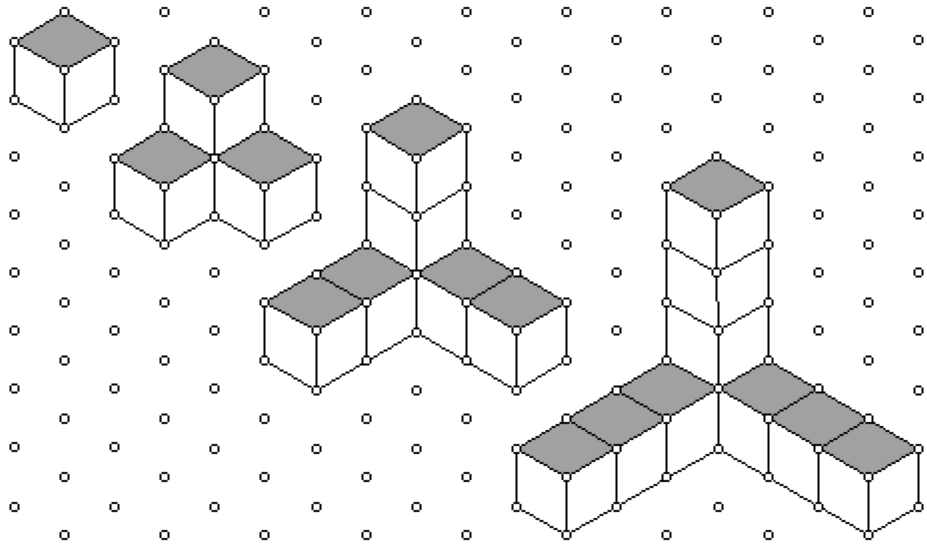
For the third model; $n = 3$. If we replace n with 3 in the formula, we can see how the 3 cubes in the second model are increased by 2 new cubes to form the third model:

$$N = 1 + (3-1) \times 2$$
$$N = 1 + (2) \times 2$$
$$N = 1 + 2 + 2$$
$$N = 3 + 2$$

g) In the formula, $N = 1 + (n-1) \times 2$. We know that the expression $(n-1)$ is connected to the Model Number, but what about the “1” and the “2”? What do these two numbers have to do with the original sequence of models?

B. ***SPROUTING ARMS***

The following sequence of models forms a definite pattern. **Describe the process** that was used to form each successive model out of the previous one. **Build the sequence of models** using cubes.



a) How many cubes would you add to the fourth model to create the fifth model? How many cubes would there be in the fifth model in this sequence?

b) Complete the following table with your prediction of the number of cubes in the fifth model and the formula that will give the number of cubes in the n^{th} model.

Model Number	1	2	3	4	5	…	n
Number of Cubes	1						

c) Will there be a model in this sequence that has exactly 71 cubes in it? **Explain your thinking.**

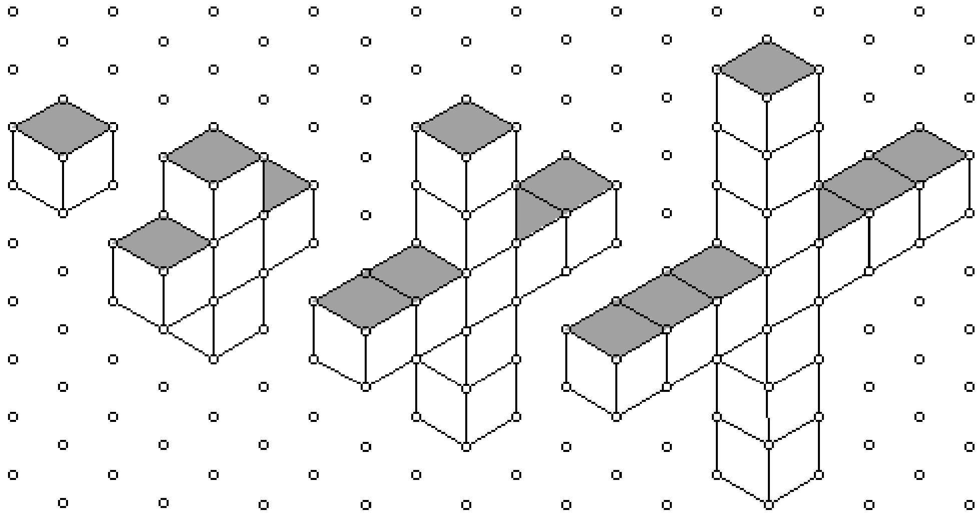
d)

> ***Think…***
> If you were to continue writing the sequence of numbers in the row titled "Number of Cubes", you will find lots of ***square numbers***: such as 1, 4, 16, 25 etc. If you have a ***square number*** of cubes you would think that you could re-assemble the model to form a square.
> What size squares is it possible to make using the numbers found in this sequence of models?

e) What is the greatest number of consecutive models that you can build using 100 cubes?

C. ***PLUS PLUSES***

The following sequence of models forms a definite pattern. **Describe the process** that was used to form each successive model out of the previous one. **Build the sequence** of models using cubes.



a) How many cubes would you add to the fourth model to create the fifth model? How many cubes would there be in the fifth model in this sequence?

b) **Complete the following table** with your prediction of the number of cubes in the fifth model and the formula that will give the number of cubes in the n^{th} model.

Model Number	1	2	3	4	5	…	n
Number of Cubes	1						

c) **Consider the surface area** of each model in this sequence of models. For each cube that is added at each step, how many **new** "faces" are added to the total surface area of the model? Be careful with this question. Make sure that you are counting only ***additional*** faces.

d) **Complete the following table** with your prediction of the number of faces on the surface of the 5^{th} and 6^{th} models. **Can you find a formula** that will give you the number of faces on the surface of the n^{th} model?

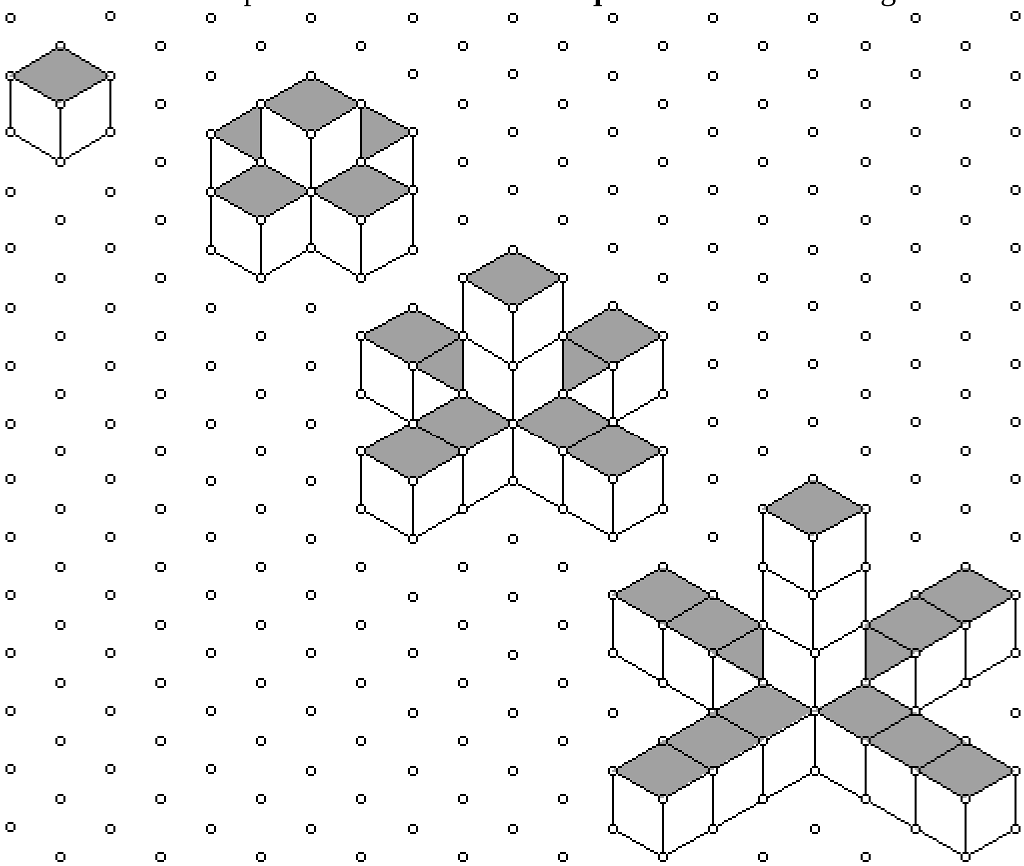
Model Number	1	2	3	4	5	6	…	n
Number of Faces	6	22						

e) **Find a formula** for the number of faces on the surface of the n^{th} model in the *Sprouting Arms*.

f) **Find a formula** for the number of faces on the surface of the n^{th} model in the *Growing "L's"*.

D. ***PARADING PEDESTALS***

The following sequence of models forms a definite pattern**. Describe the process** that was used to form each successive model out of the previous one. **Build the sequence of models** using cubes.



a) **Complete the following table** with your prediction of the number of cubes in the fifth model and the formula that will give the number of cubes in the n^{th} model.

Model Number	1	2	3	4	5	…	n
Number of Cubes	1	6					

b

Think…

If you were to continue writing the sequence of numbers in the row titled "Number of Cubes", you will find lots of ***square numbers***: such as 1, 4, 16, 25 etc. If you have a ***square number*** of cubes you would think that you could re-assemble the model to form a square.

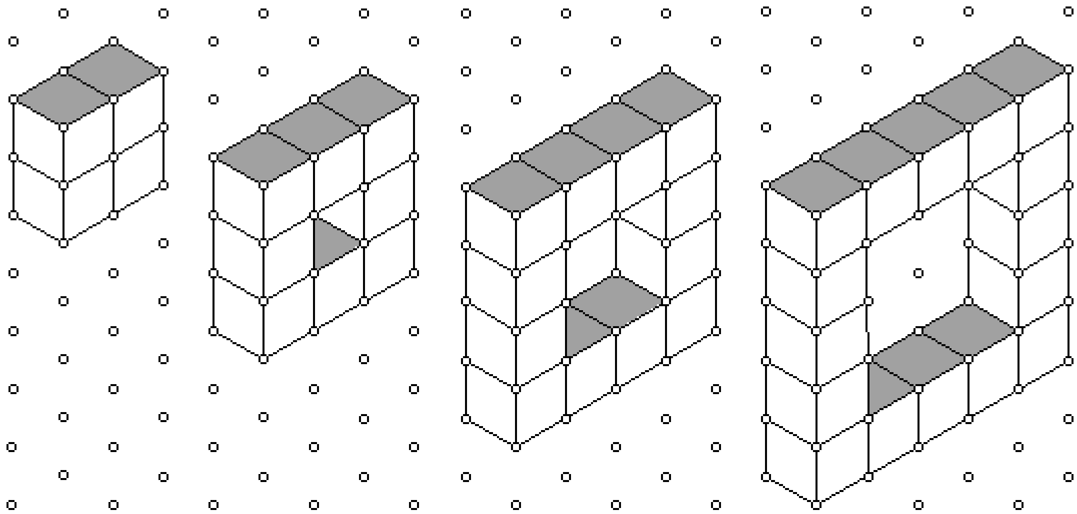
What size squares is it possible to make using the numbers found in this sequence of models?

c) **Complete the following table** with your prediction of the number of faces on the surface of the 5^{th} and 6^{th} models. **Can you find a formula** that will give you the number of faces on the surface of the n^{th} model?

Model Number	1	2	3	4	5	6	…	n
Number of Faces	6	26						

E. ***PICTURE THIS***

The following sequence of models forms a definite pattern. **Describe the process** that was used to form each successive model out of the previous one. **Build the sequence** of models using cubes.



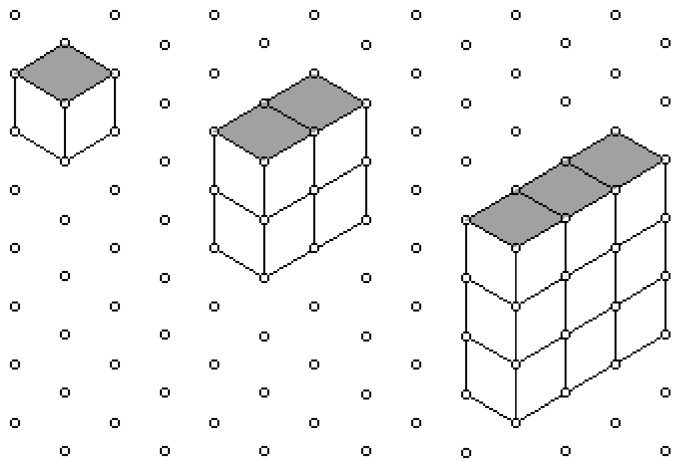
a) **Complete the following table** with your prediction of the number of cubes in the fifth model and the formula that will give the number of cubes in the n^{th} model.

Model Number	1	2	3	4	5	…	n
Number of Cubes	4						

b) **Complete the following table** with your prediction of the number of faces on the surface of the 5^{th} and 6^{th} models. **Can you find a formula** that will give you the number of faces on the surface of the n^{th} model?

Model Number	1	2	3	4	5	6	…	n
Number of Faces	16							

The above models could be described as "squares with square holes". You can see how the squares shown in the sequence below might "fill the holes". The first square below could "fill" the hole in the second model above. The second square below could "fill" the hole in the third model above. And so on.



c)

Think:
Explain why the following equation has to be true for all integer values of $n \geq 1$.

$$4n = (n+1)^2 - (n-1)^2$$

Use both algebra and cubes in your explanation.

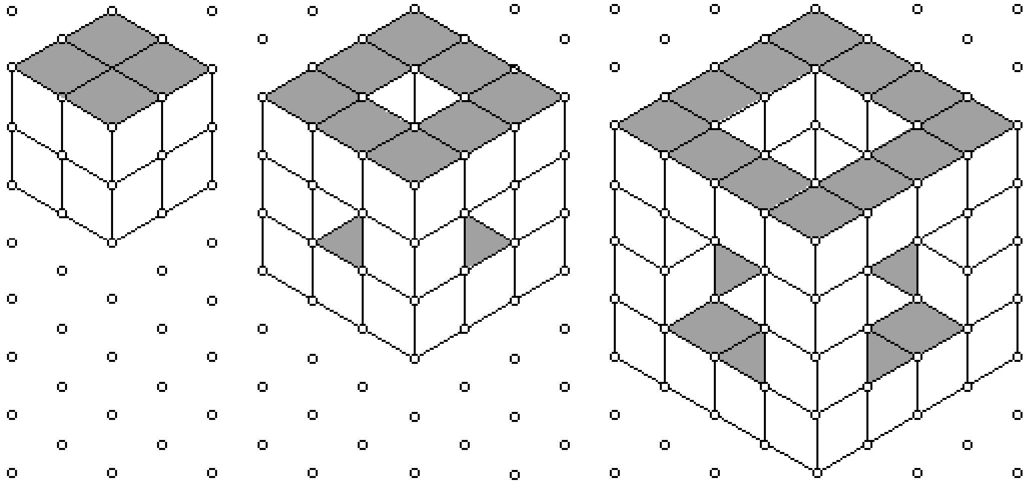
i) What does n represent in this equation?

ii) What does $(n+1)$ represent in this equation?

iii) What does $(n-1)$ represent in this equation?

F. ***STROLLING BONES***

The following sequence of models forms a definite pattern. Describe the process that was used to form each successive model out of the previous one. Build the sequence of models using cubes.



a) **Complete the following table** with your prediction of the number of cubes in the fourth and fifth model and the formula that will give the number of cubes in the n^{th} model.

Model Number	1	2	3	4	5	…	n
Number of Cubes	8						

b) A student has constructed a model that uses 356 cubes. Which model is this and how many cubes are there in each edge?

c)

> ***Think…***
> If you were to write down the numbers of cubes in the first 10 or so models, you will notice that the numbers in the odd positions are all multiples of 8. Describe this pattern more fully and give an explanation as to why it is there.

d)

> ***Think…***
> You will find that if you disassemble all of the skeleton cubes, some of them can be re-assembled into solid cubes.
> The first model happens to be a solid cube already. The 43rd model in this sequence is the next model that can be disassembled and re-assembled into a solid cube.
> The 43rd model contains 512 cubes. $512 = 8^3$.
> Using a calculator, try to find other models in this sequence that have this property.
> Explain the strategy that you used to help you find these interesting skeleton cubes.

e) **Complete the following table** with your prediction of the number of faces on the surface of the 4^{th} and 5^{th} models. **Can you find a formula** that will give you the number of faces on the surface of the n^{th} model?

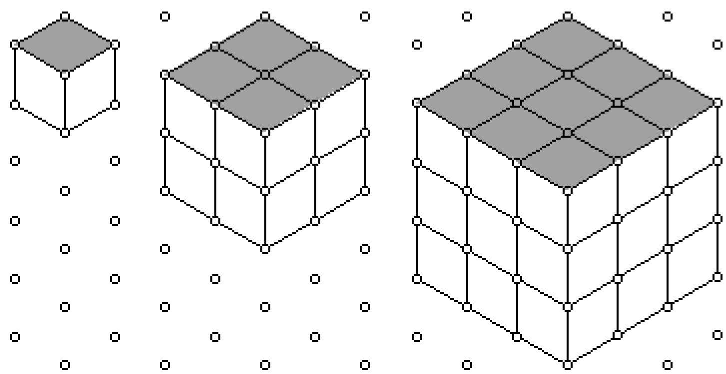
Model Number	1	2	3	4	5	…	n
Number of Faces	24	72					

f) A student has been using the formula *number of faces* $= 24\times(2n-1)$. Is this correct? **Explain why or why not**.

g) Which model will have a surface area 15 times as large as the first model's?

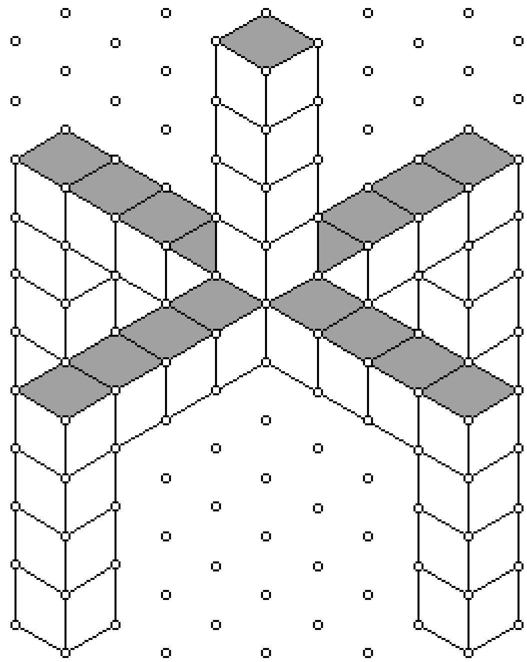
h) **Determine the ratio** of *Surface Area* to *Volume* for these models. What happens to this ratio as the size of the model increases?

i) **Determine the ratio** of *Surface Area* to *Volume* for the following models. What happens to this ratio as the size of the model increases?



G. ***DADDY LONGLEGS***

The following figure is one in a series. The previous model was made of 24 cubes.



a) How many cubes will there be in the next model? **Explain your thinking**.

b)

> *Think…*
> How many cubes are there in the first model in this sequence? **Explain your thinking**.

c) **Determine a formula** for the number of cubes in the n^{th} model in this sequence.